VirtualLab Fusion Technology – Solvers and Functions

Local Linear Grating Approximation (LLGA)

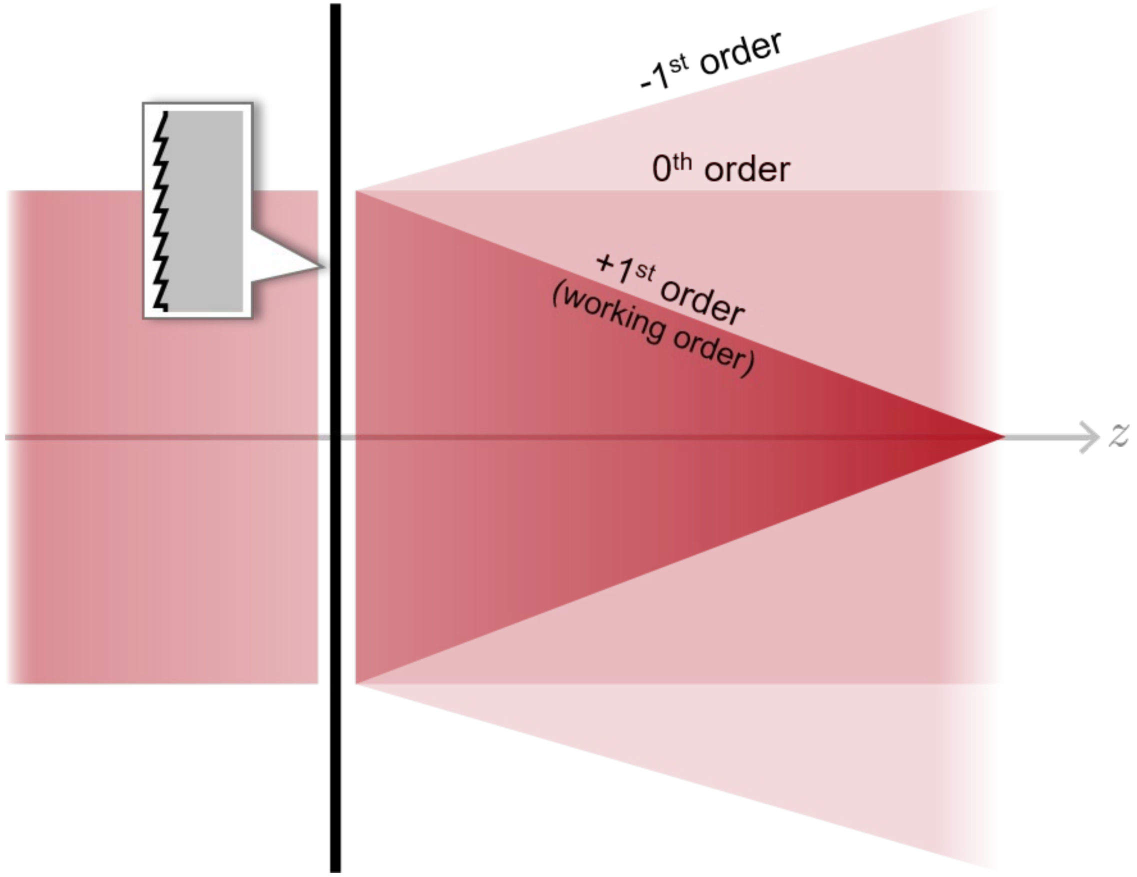
For the **Diffractive Lens Component, Holographic Optical Element Component**

Abstract

The LLGA solver works in the spatial domain (**x domain**), locally, in a pointwise manner. The solver follows that

1. the input field on the surface is treated as a composition of local plane waves (LPWs),
2. the part of the surface seen by each LPW is considered a linear grating (locally), and,
3. the interaction of the LPW with the local linear grating can be modeled by the FMM/RCWA, TEA, or the idealized grating function.

At an arbitrary location on the surface, an approximate local boundary condition is applied, which assumes the interaction of the LPW with the local linear grating. Thus, the FMM/RCWA, TEA, or the idealized grating function can respectively be used to connect input and output fields.



Solver Algorithm

- Both the input and output fields are defined on a **reference surface** (which is usually a plane at $z = 0$), in the form of

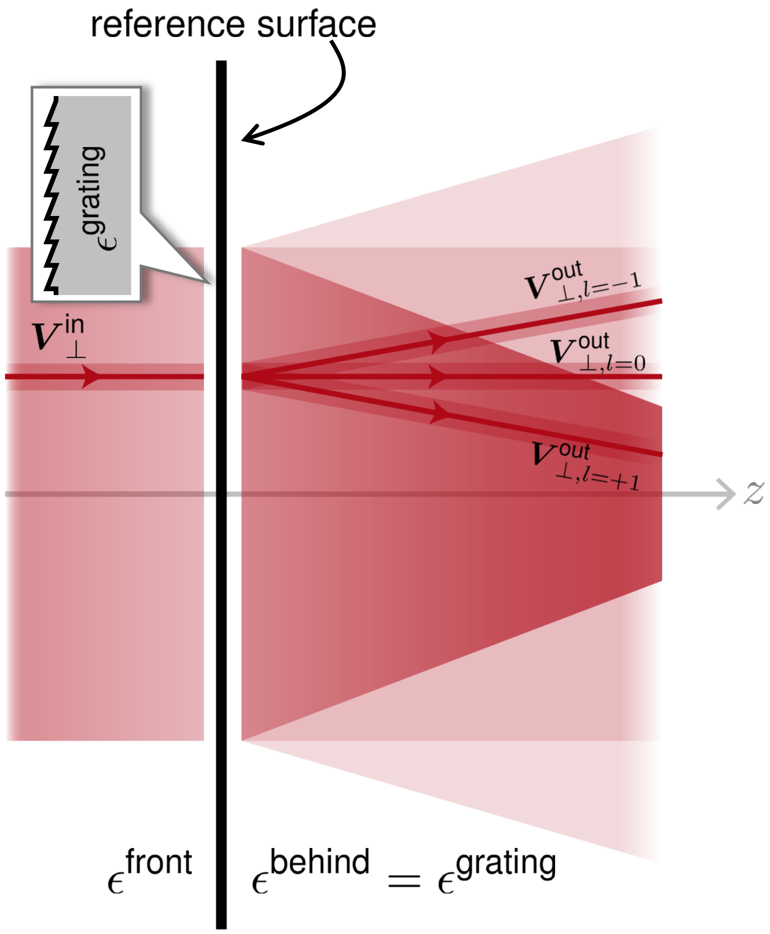
$$\boldsymbol{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) = \boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\right) ,$$
$$\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\text{out}}(\boldsymbol{\rho})\right) ,$$

  where ψ is the wavefront phase part, $\boldsymbol{U}_{\perp}$ is the residual fields, with $\boldsymbol{\rho} = (x, y)$ as the transverse coordinates.

- The output field is to be calculated, according to the **diffraction order index** l, pointwisely in the x domain, as

$$\boldsymbol{V}_{\perp,l}^{\text{out}}(\boldsymbol{\rho}) = \mathbf{B}_l(\boldsymbol{\rho})\boldsymbol{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) ,$$

  or, explicitly, with
  - the redisual field: $\boldsymbol{U}_{\perp,l}^{\text{out}}(\boldsymbol{\rho}) = \mathbf{b}_l(\boldsymbol{\rho})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$, and
  - the wavefront phase part: $\psi_l^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(\boldsymbol{\rho}) + l\Delta\psi^{\text{D}}(\boldsymbol{\rho})$, where $\Delta\psi^{\text{D}}$ is the **design phase modulation** for the working order.



Solver Algorithm

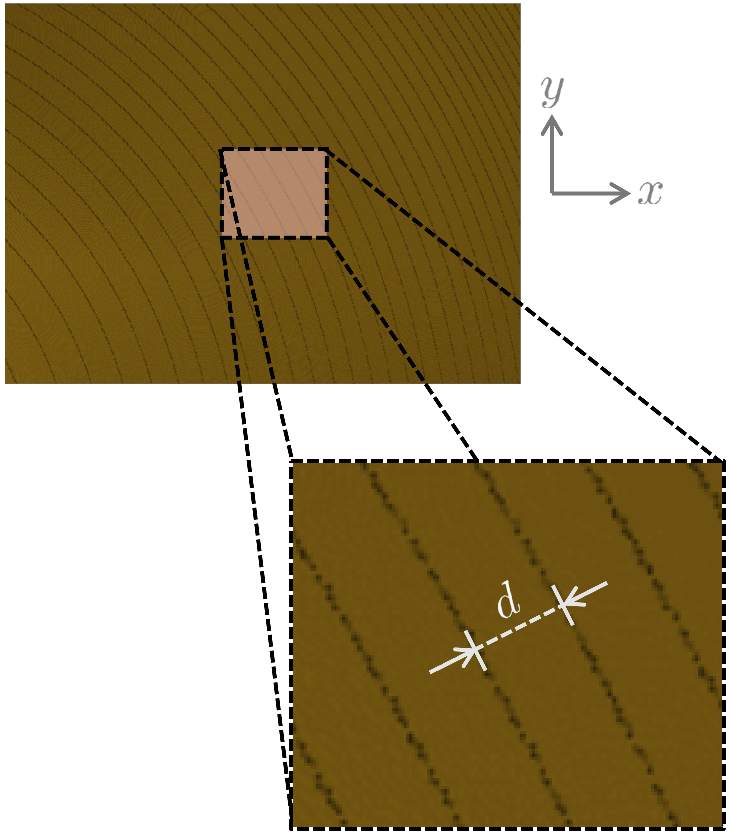
- The B matrix value is to be determined from the local linear grating at the location $\boldsymbol{\rho}$.
- In practice, the local grating information is usually specified implicity via the **design phase modulation** $\Delta\psi^{\mathsf{D}}(\boldsymbol{\rho})$. For example, the local grating period can be found as follows
  - calculate local grating period projection components

$$d_x(\boldsymbol{\rho}) = \frac{2\pi}{\partial_x \Delta\psi^{\mathsf{D}}(\boldsymbol{\rho})}\,, \quad d_y(\boldsymbol{\rho}) = \frac{2\pi}{\partial_y \Delta\psi^{\mathsf{D}}(\boldsymbol{\rho})}\,;$$

  - then determine the local grating period

$$d(\boldsymbol{\rho}) = \sqrt{d_x^2(\boldsymbol{\rho}) + d_y^2(\boldsymbol{\rho})}\,,$$

with its orientation along $\hat{\boldsymbol{p}} = \frac{1}{d}(d_x, d_y)$.

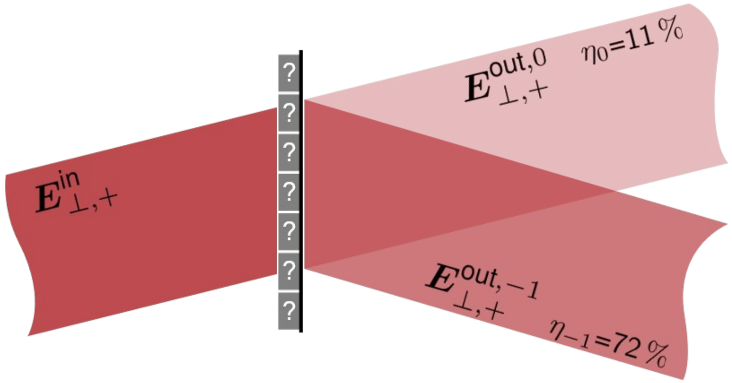


Solver Algorithm

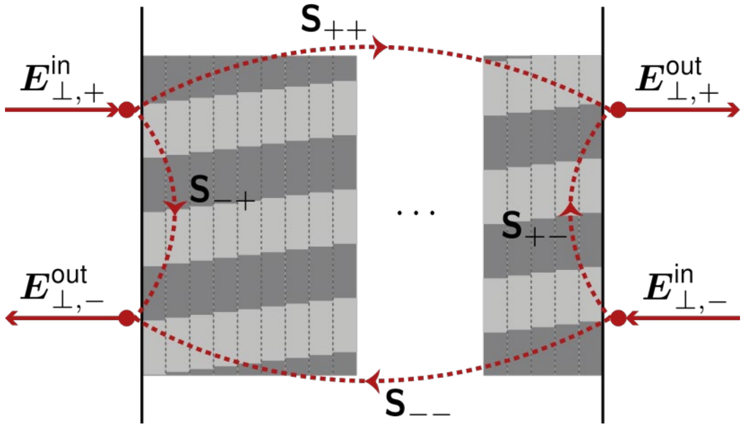
- The local grating period and orientation fixes the diffraction directions, while the field-related property is to be determined.
- The field-related diffraction property, that is specified by the matrix $\mathbf{b}_l(\boldsymbol{\rho})$, can be calculated by, either
  - using the **idealized grating function**, or
  - using FMM/RCWA as rigorous the solver, or thin element approximation (TEA), for **actual grating structures**.
- The actual grating profile, when needed, is implicitly determined via the design phase modulation $\Delta\psi^{\mathrm{D}}(\boldsymbol{\rho})$, as

$$h(\boldsymbol{\rho}) = \alpha \frac{\Delta\psi^{\mathrm{D}}}{2\pi\Delta n}\lambda^{\mathrm{D}}\,,$$

with λ^{D} as the design wavelength in vacuum, n^{D} as the design refractive index difference, and α as the scaling factor.



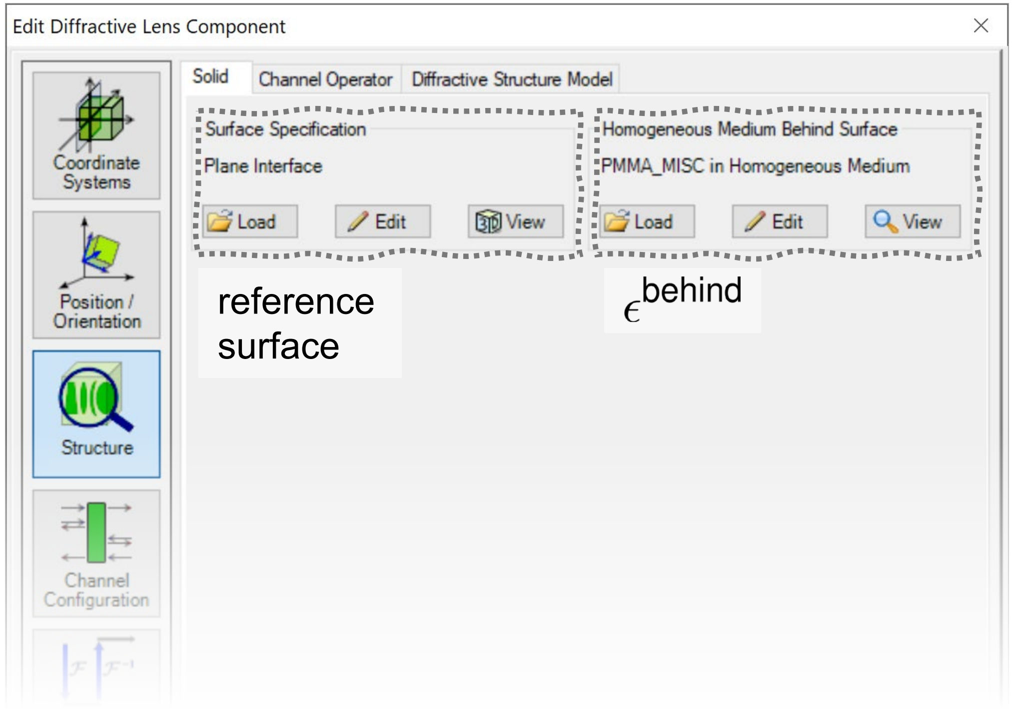
idealized grating function



FMM / RCWA

Usage in VirtualLab Fusion

- Take the Diffractive Lens Component as an example:
  - the **reference surface** is specified under the Structure tab and a plane surface is used by default;
  - one can define $\epsilon^{\textbf{behind}}$ with the homogeneous medium behind the surface, while $\epsilon^{\textbf{front}}$ is automatically determined by the preceding system;
  - the permittivities ϵ^{front} and $\epsilon^{\text{behind}}(= \epsilon^{\text{grating}})$ also defines the design refractive index difference, as $\Delta n^{\text{D}} = |\sqrt{\epsilon^{\text{front}}} - \sqrt{\epsilon^{\text{behind}}}|$, which is needed to determine the actual grating structure.

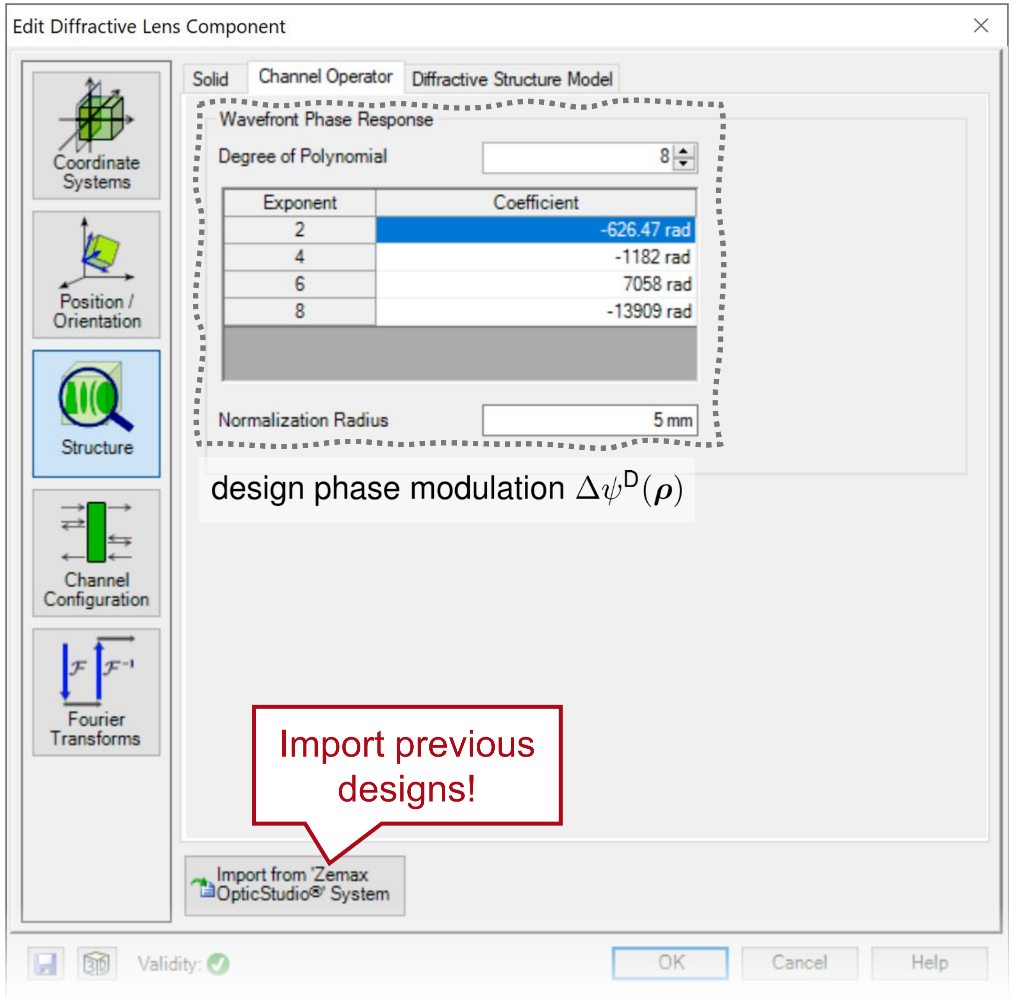


Surface Specification: Plane Interface

Homogeneous Medium Behind Surface: PMMA_MISC in Homogeneous Medium

Usage in VirtualLab Fusion

- Take the Diffractive Lens Component as an example:
  - the **design phase modulation** $\Delta\psi^{\mathrm{D}}(\boldsymbol{\rho})$ is specified, in this case, by a rotational symmetric polynomial, with even orders;
  - this is a compatible definition with the Binary 2 surface in Zemax OpticStudio® and one may import the previous designs into VirtualLab Fusion.



Degree of Polynomial: 8

Exponent	Coefficient
2	-626.47 rad
4	-1182 rad
6	7058 rad
8	-13909 rad

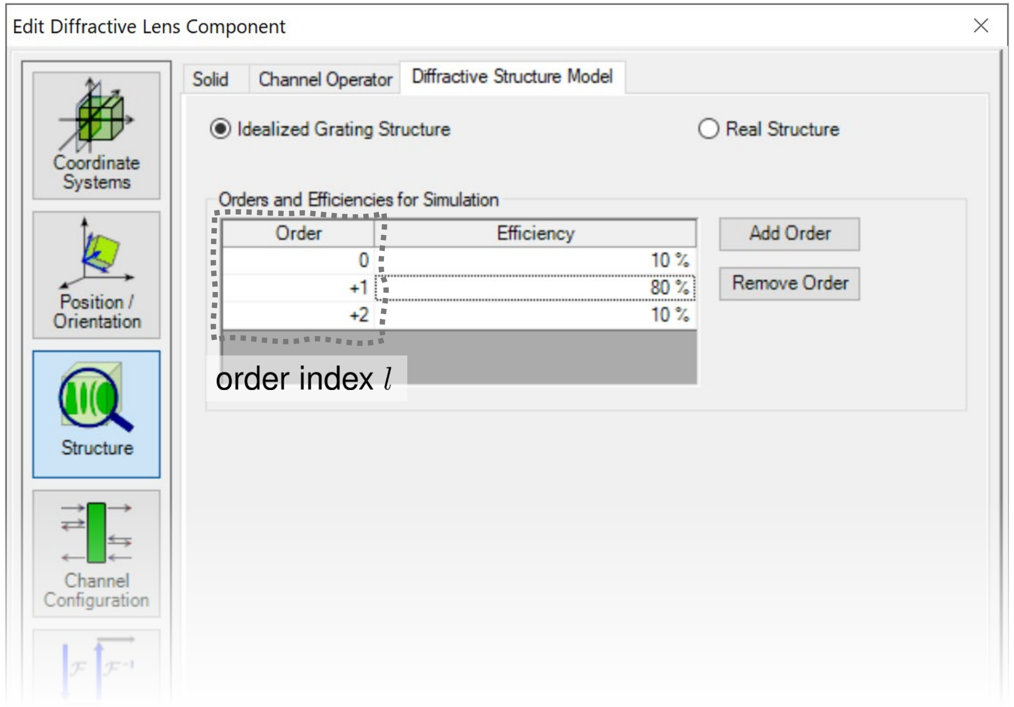
Normalization Radius: 5 mm

design phase modulation $\Delta\psi^{\mathrm{D}}(\boldsymbol{\rho})$

Import previous designs!

Usage in VirtualLab Fusion

- Take the Diffractive Lens Component as an example:
    - for the field-related diffraction property, one can choose between **idealized grating function** and FMM/RCWA or TEA for actual grating structrue modeling;
    - with the idealized grating fuction, the indices of the diffraction orders under investigation shall be specified, together with the desired efficiencies;



● Idealized Grating Structure ○ Real Structure

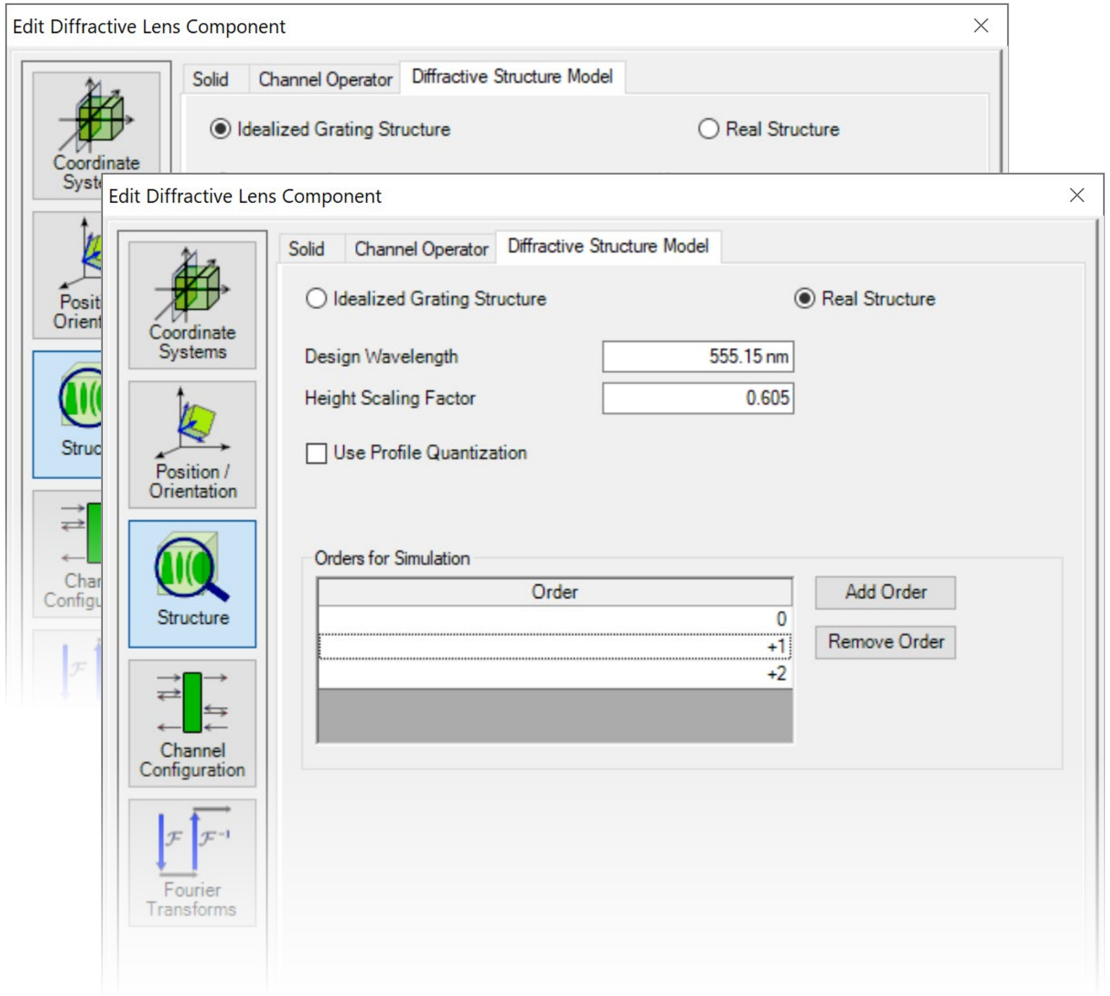
Orders and Efficiencies for Simulation

Order	Efficiency
0	10 %
+1	80 %
+2	10 %

order index l

Usage in VirtualLab Fusion

- Take the Diffractive Lens Component as an example:
    - for the field-related diffraction property, one can choose between idealized grating function and FMM/RCWA or TEA for **actual grating structrue** modeling;
    - with the idealized grating fuction, the indices of the diffraction orders under investigation shall be specified, together with the desired efficiencies;
    - for the actual grating structure, the design wavelength and the scaling factor shall be specified, and one can select the indices of the diffraction orders under investigation as well.



Edit Diffractive Lens Component

● Idealized Grating Structure ○ Real Structure

Edit Diffractive Lens Component

○ Idealized Grating Structure ● Real Structure

Design Wavelength: 555.15 nm

Height Scaling Factor: 0.605

☐ Use Profile Quantization

Orders for Simulation

Order
0
+1
+2

Document Information

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further reading	- VirtualLab Fusion Technology – Local Plane Interface Approximation (LPIA) - VirtualLab Fusion Technology – FMM / RCWA [S-Matrix] - VirtualLab Fusion Technology – Idealized Grating Functions - Design and Analysis of Intraocular Diffractive Lens - Modeling of a Hybrid Eyepiece with Diffractive Lens Surface for Chromatic Aberration Correction